Key

Math 4 Name______________________

1-7 Summation Notation Date________

paradox: *a statement, proposition, or situation that seems to be absurd or contradictory, but in fact is or may be true*

The Greek philosopher Zeno of Elea lived around 450 B.C and is now famous for his paradoxes. One of them involves a runner who is trying to go from point A to point B. Zeno pointed out that the runner would first have to go half the distance, then half the remaining distance, and so forth. Suppose the runner travels at a constant speed, taking one minute to go half the distance from A to B. It will then take $\frac{1}{2}$ minute to go half of the remaining distance, $\frac{1}{4}$ minute to go half of the new remaining distance, and so forth.

Zeno observed that the total time required for the runner to reach point B would have to equal the sum of an *infinite* collection of positive numbers:

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$$

Summation notation, or *sigma notation*, is very convenient for concisely representing sums such as that arising from this paradox of Zeno. This notation uses the symbol $\sum$ (the Greek capital letter sigma).

The infinite sum above could be written as $\sum_{i=1}^{\infty}\frac{1}{2^{i-1}}$

Definition of Summation Notation

$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + a_{m+2} + \ldots a_n$$

The symbol $\sum_{i=m}^{n} a_i$ *is read "the sum of the numbers* a_i *from* $i = m$ *to* $i = n$."

Writing $\sum_{i=m}^{n} a_i$ *as* $a_m + a_{m+1} + a_{m+2} + \ldots a_n$ *is called* ***expanded form***

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1. Write $\sum_{i=1}^{4} 3i$ in expanded form and find the value of this sum.

$= 3(1) + 3(2) + 3(3) + 3(4)$

$= 3 + 6 + 9 + 12 = \boxed{30}$

2. Write $\sum_{n=2}^{6} 2n^2$ in expanded form and find the value of this sum.

$= 2(2)^2 + 2(3)^2 + 2(4)^2 + 2(5)^2 + 2(6)^2$

$= 8 + 18 + 32 + 50 + 72$

$= \boxed{180}$

3. Consider the summation $\sum_{k=1}^{n} \frac{n+1}{n+k}$

a. Write the expanded form of the summation for $n = 1$ $\rightarrow$ $\sum_{k=1} \frac{n+1}{n+k}$

$$= \frac{1+1}{1+1} = \frac{2}{2} = \boxed{1}$$

b. Write the expanded form of the summation for $n = 4$ and approximate its value.

$$\sum_{k=1}^{4} \frac{n+1}{n+k} = \frac{4+1}{4+1} + \frac{4+1}{4+2} + \frac{4+1}{4+3} + \frac{4+1}{4+4}$$

$$= \frac{5}{5} + \frac{5}{6} + \frac{5}{7} + \frac{5}{8} = \boxed{\frac{533}{168} \approx 3.173}$$

4. Write the following sums in summation notation. You do NOT have to find the sum.

a. $2+4+6+8+10+...$

Arith. $d=2$

$$\boxed{\sum_{i=1}^{n} 2+2(i-1)}$$

b. $\overset{1}{1}+\overset{2}{4}+9+16+25+36+49+\overset{8}{64}$

$$\boxed{\sum_{k=1}^{8} k^2}$$

c. $3+6+12+24+48+96+...$

Geo. $r=2$

$$\boxed{\sum_{j=1}^{n} 3(2)^{j-1}}$$

d. $3+7+11+15+19+23$

Arith. $d=4$

$$\boxed{\sum_{i=1}^{6} 3+4(i-1)}$$

5. Expand the first several terms of the following summation and then attempt to find the sum:

$$\sum_{n=1}^{100} n$$

$1+2+3+4+ \dots +99+100$

$$\text{Sum} = \frac{n}{2}(a_1 + a_n) = \frac{100}{2}(1+100) = 50(101)$$

$$= \boxed{5050}$$